

Apex 2014 Workshop

Measure the Ring Transfer and Coupling
Matrix at the IPs

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Uncoupled Motion

$$M_{1,2} = \begin{bmatrix} \cos(\mu_{1,2}) + \alpha_{1,2} \sin(\mu_{1,2}) & \beta_{1,2} \sin(\mu_{1,2}) \\ -\frac{1+\alpha_{1,2}^2}{\beta_{1,2}} \sin(\mu_{1,2}) & \cos(\mu_{1,2}) - \alpha_{1,2} \sin(\mu_{1,2}) \end{bmatrix}$$

$$U = \begin{bmatrix} M_1 & \mathbf{0} \\ \mathbf{0} & M_2 \end{bmatrix}$$

	Quadrant II 0 > cos($\mu_{1,2}$) 0 < sin($\mu_{1,2}$)	Quadrant I 0 < cos($\mu_{1,2}$) 0 < sin($\mu_{1,2}$)
$\mu_{1,2} = 2\pi Q_{1,2}$	Quadrant III 0 > cos($\mu_{1,2}$) 0 > sin($\mu_{1,2}$)	Quadrant IV 0 < cos($\mu_{1,2}$) 0 > sin($\mu_{1,2}$)

Coupled Motion

Coupled motion used in MADX. This is different from Edwards and Teng.

$$G = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \bar{G} \equiv S G^T S^T = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \quad S \equiv \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$H = \frac{1}{\sqrt{1+det(G)}} \begin{bmatrix} I & \bar{G} \\ -G & I \end{bmatrix} \quad H^{-1} = \frac{1}{\sqrt{1+det(G)}} \begin{bmatrix} I & -\bar{G} \\ G & I \end{bmatrix}$$

$$T = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = H U H^{-1} = H \begin{bmatrix} M_1 & \mathbf{0} \\ \mathbf{0} & M_2 \end{bmatrix} H^{-1}$$

$$[Q_1, \alpha_1, \beta_1, Q_2, \alpha_2, \beta_2, a, b, c, d]$$

Coupled Motion

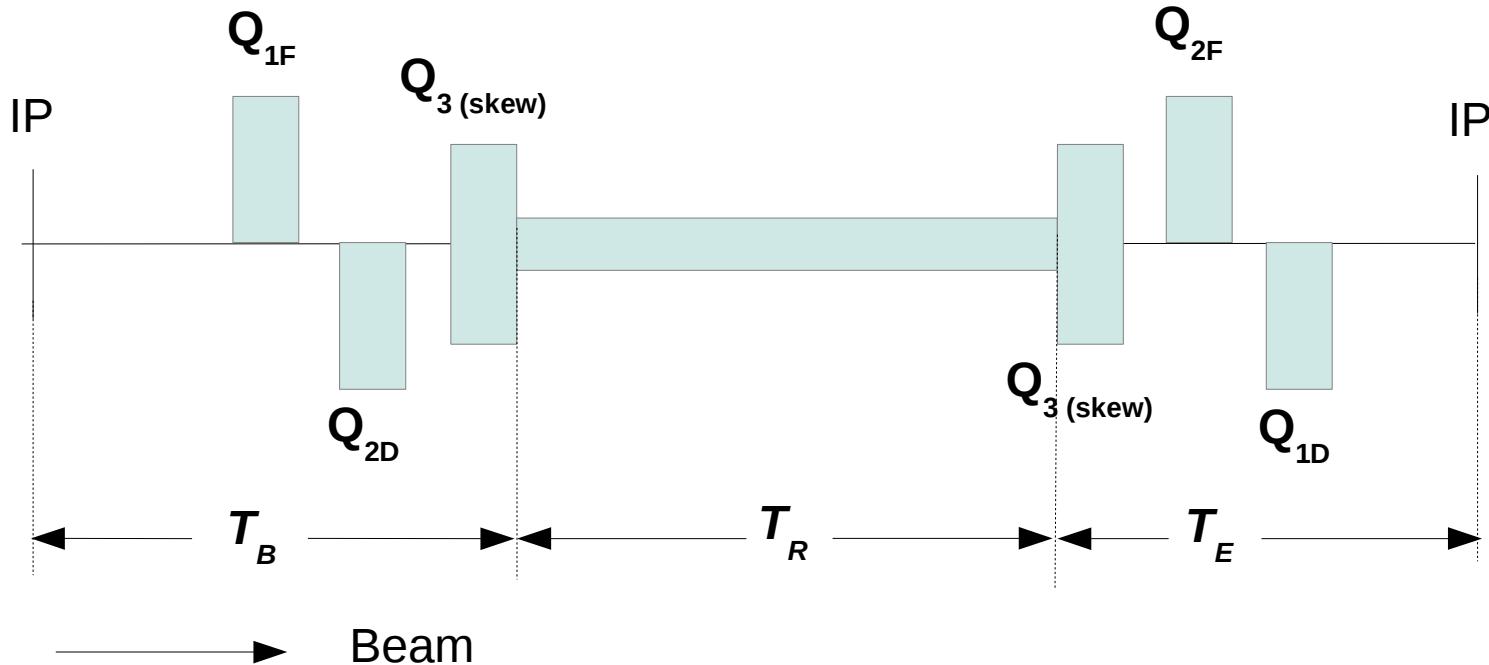
$$\mathbf{G} = \frac{1 + \det(\mathbf{G})}{\text{Tr}(\mathbf{M}_2) - \text{Tr}(\mathbf{M}_1)} (\bar{\mathbf{B}} + \mathbf{C}) = - \left[\frac{1}{2} (\text{Tr}(\mathbf{A}) - \text{Tr}(\mathbf{D})) \pm \sqrt{\frac{1}{4} (\text{Tr}(\mathbf{A}) - \text{Tr}(\mathbf{D}))^2 + \det(\bar{\mathbf{B}} + \mathbf{C})} \right]^{-1} (\bar{\mathbf{B}} + \mathbf{C})$$

$$\mathbf{M}_1 = \mathbf{A} - \bar{\mathbf{G}} \quad \mathbf{C} = \mathbf{A} - \mathbf{B} \mathbf{G} \quad \mathbf{M}_2 = \mathbf{D} + \mathbf{G} \quad \mathbf{B} = \mathbf{D} + \mathbf{C} \bar{\mathbf{G}}$$

$$Q_{1,2} = \text{Tune}_{1,2}(\mathbf{T}) = \text{Tune}(\mathbf{M}_{1,2}) = \begin{cases} \frac{\arccos(\text{Tr}(\mathbf{M}_{1,2})/2)}{2\pi} & (\mathbf{M}_{1,2})_{12} > 0 \\ 1 - \frac{\arccos(\text{Tr}(\mathbf{M}_{1,2})/2)}{2\pi} & (\mathbf{M}_{1,2})_{12} < 0 \end{cases}$$

$$\Delta Q_{min} = D\text{tuneMin}(\mathbf{T}) = \frac{\pm \sqrt{\det(\bar{\mathbf{B}} + \mathbf{C})}}{\pi [\sin(2\pi Q_A) + \sin(2\pi Q_D)]} \quad Q_A = \text{Tune}(\mathbf{A}) \quad Q_D = \text{Tune}(\mathbf{D})$$

Applying the Method



$$T = T_E T_R T_B$$

$$T_{\Delta_i} = T_{E+\Delta_i} T_R T_B = T_{E+\Delta_i} (T_E^{-1} T_E) T_R T_B = (T_{E+\Delta_i} T_E^{-1}) T_E T_R T_B = (T_{E+\Delta_i} T_E^{-1}) T$$

$$T_{\Delta_i} = T_E T_R T_{B+\Delta_i} = T_E T_R (T_B T_B^{-1}) T_{B+\Delta_i} = T_E T_R T_B (T_B^{-1} T_{B+\Delta_i}) = T (T_B^{-1} T_{B+\Delta_i})$$

Error Analysis

$$\vec{z} = (\alpha_1, \beta_1, \alpha_2, \beta_2, a, b, c, d).$$

$$\chi^2(\vec{z}) = \sum_{i=1}^{19} \left(\frac{Q_i - q_i(\vec{z})}{\delta Q_i} \right)^2$$

$$\vec{g} \equiv -\frac{1}{2} \nabla_{\vec{z}} [\chi^2] = \sum_{i=1}^{19} \frac{Q_i - q_i(\vec{z})}{\delta Q_i^2} \nabla_{\vec{z}} q_i(\vec{z})$$

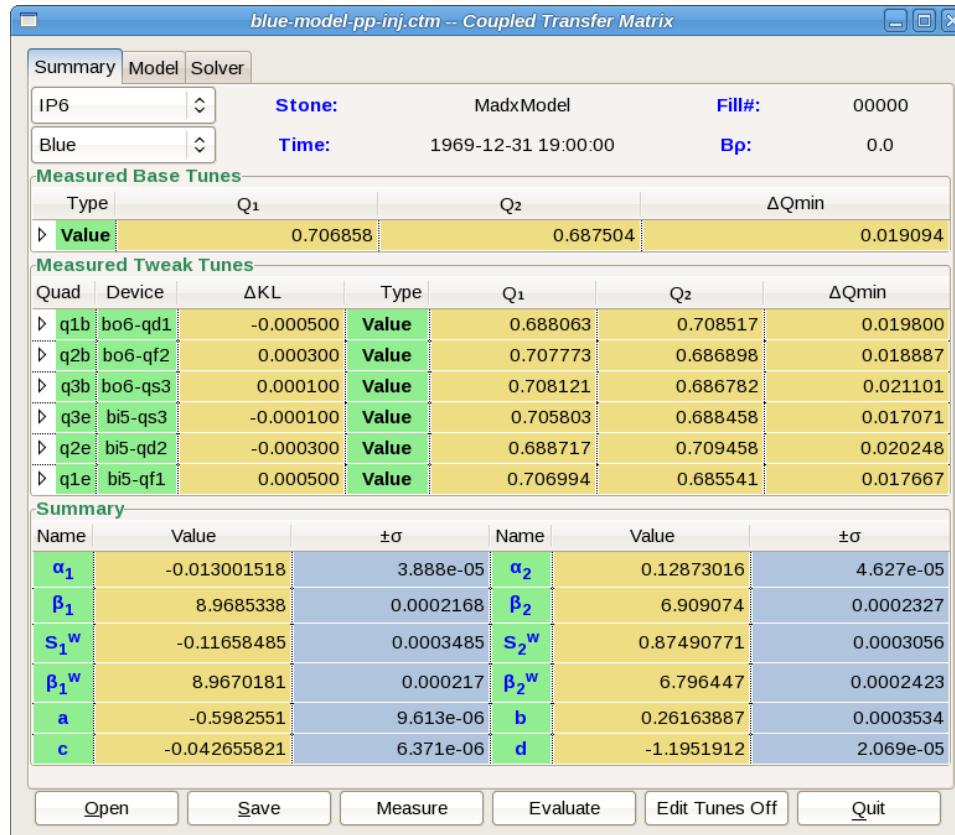
$$\mathbf{H} \equiv \frac{1}{2} \nabla_{\vec{z}} \nabla_{\vec{z}} [\chi^2] = \sum_{i=1}^{19} \frac{1}{\delta Q_i^2} \left[\nabla_{\vec{z}} q_i(\vec{z}) \circ \nabla_{\vec{z}} q_i(\vec{z}) - (Q_i - q_i(\vec{z})) \nabla_{\vec{z}} \nabla_{\vec{z}} q_i(\vec{z}) \right]$$

$$C = \mathbf{H}^{-1} \quad \quad \delta z_i = \sqrt{\frac{\chi^2(\vec{z})}{11} C_{ii}}$$

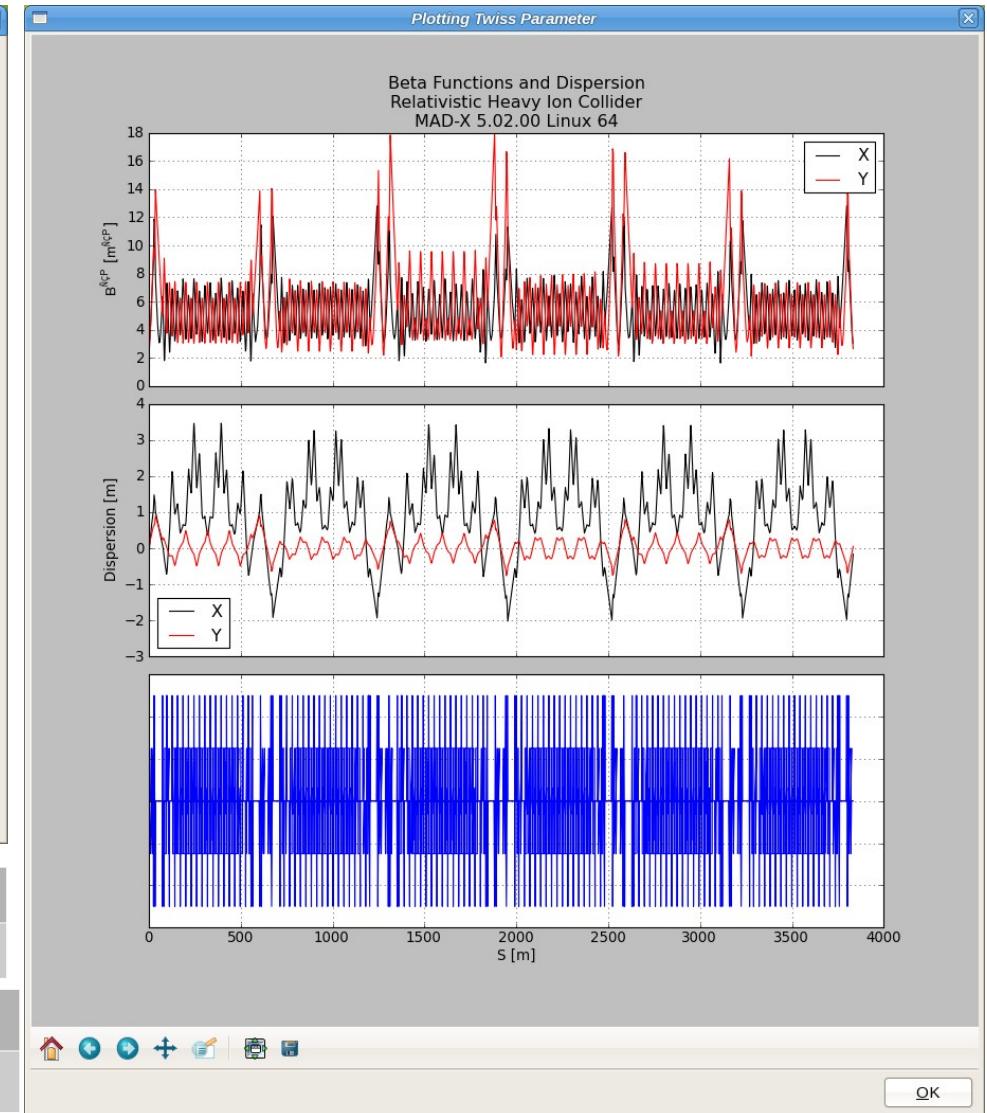
11 = 19 (measurements) – 8 (parameters)

From: W. H. Press, et al, Numerical Recipes

Simulation



BETX [m]	ALFX	BETY [m]	ALFY
8.96821989	-0.01297529327	6.908732319	0.1286595191
R11	R12	R21	R22
-0.5982619207	0.2619994028	-0.04266332603	-1.195177856



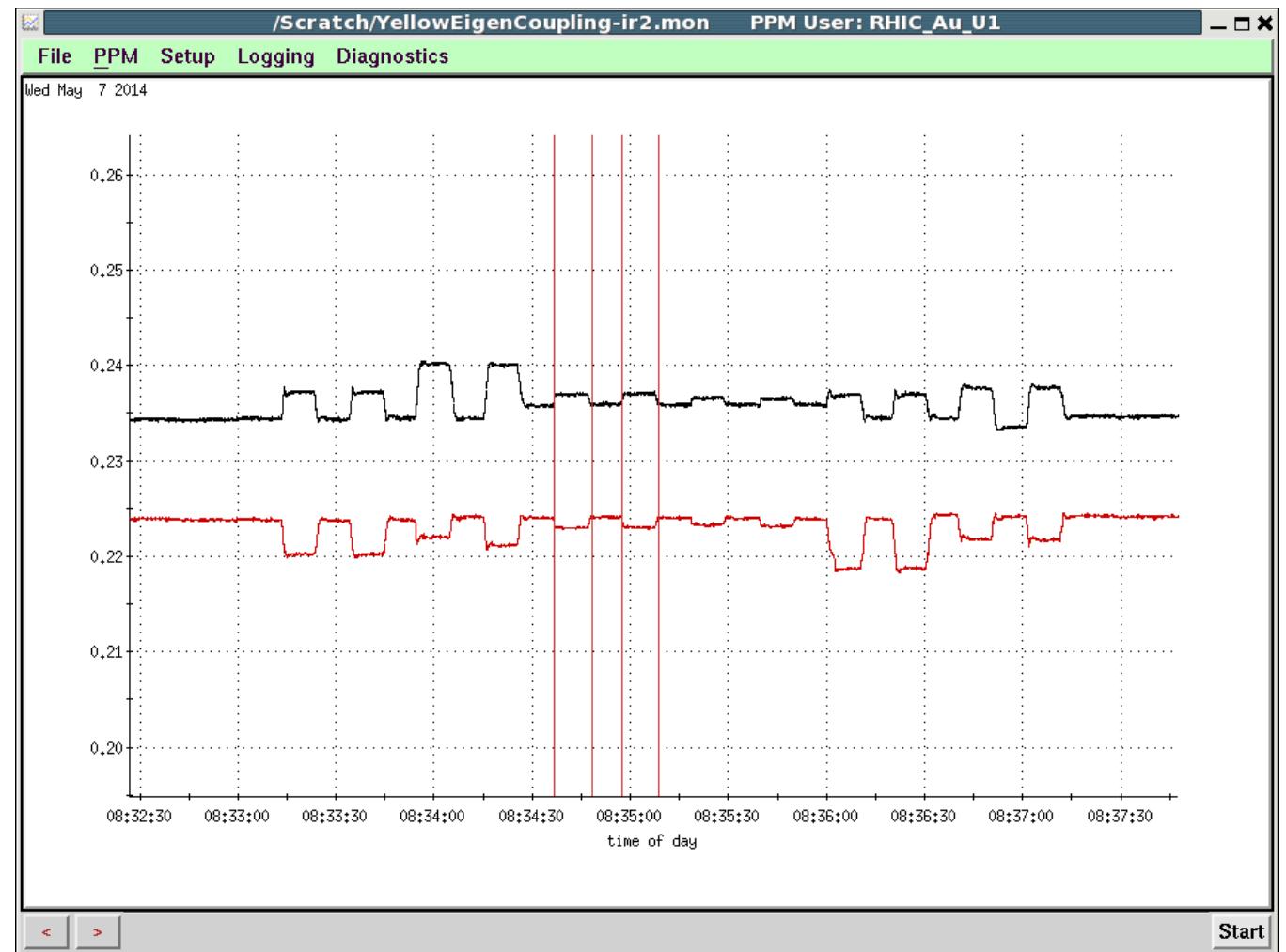
Experimental Results

Early attempts:

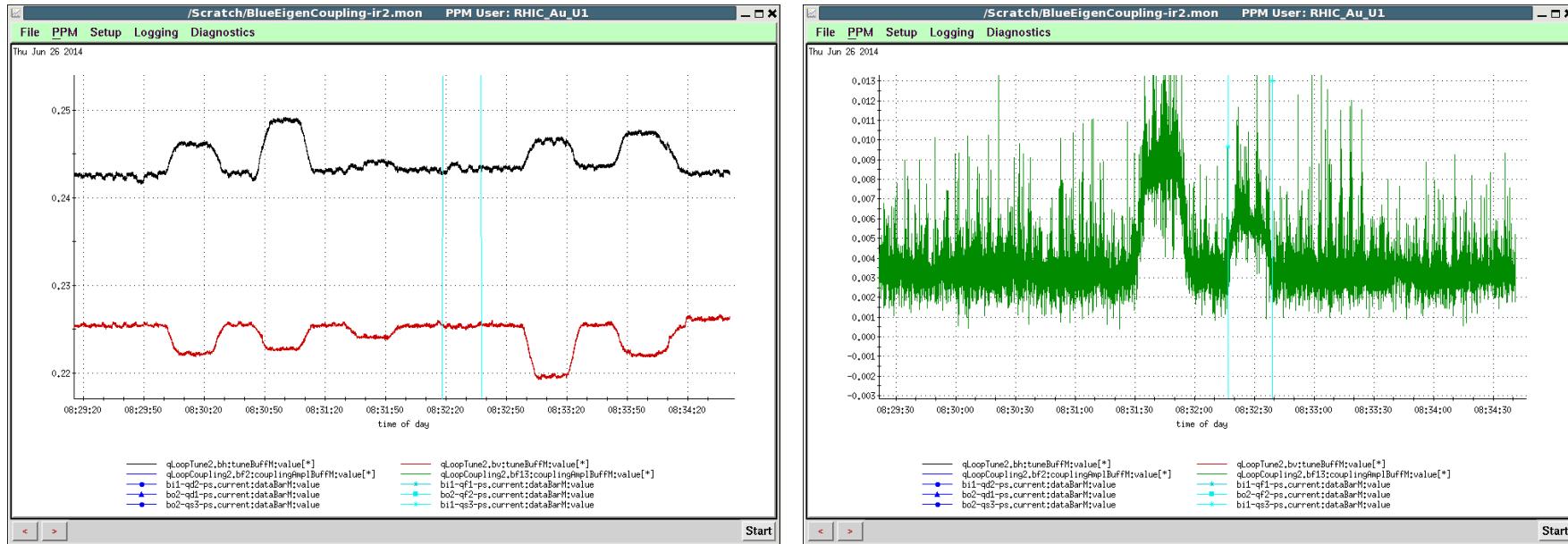
Tunes would not return to their previous states after turning off the quadrupole tweaks.

Slowfactor 10 was used on later runs.

Added Automation.



Experimental Results



Type	Q_1	$\pm\sigma$	Q_2	$\pm\sigma$	ΔQ_{min}	$\pm\sigma$
Base	0.243070	0.000413	0.225433	0.000123	0.003305	0.000613
q1b	0.246707	0.000152	0.222179	0.000116	0.003340	0.000717
q2b	0.249044	0.000137	0.249044	0.000107	0.003138	0.000761
q3b	0.244068	0.000158	0.224096	0.000108	0.009271	0.001084
q3e	0.243327	0.000351	0.225337	0.000136	0.006208	0.000758
q2e	0.246236	0.000230	0.219609	0.000135	0.003426	0.000835
q1e	0.246890	0.000131	0.222018	0.000095	0.003050	0.000532

Experimental Results

The columns “Simplex” through “SLSQP” shows the achieved χ^2 using some of the solvers available from <http://scipy.org/> (version, 0.7.2). These cases used IP2 since, there is no experimental solenoids at this IP for modeling.

Case	Ring	Fill#	Species	IP	Simplex	Lmdif	BFGS	Powell	CG	TNC	SLSQP
1	Blue	18481	^3He	IP2	3.511	3.997	3.997	3.997	3.997	3.997	3.997
2	Blue	18481	^3He	IP2	8.624	8.624	12.076	8.624	12.076	12.076	8.624
3	Blue	18481	^3He	IP2	13.662	13.662	13.662	16.478	13.662	13.663	13.662
4	Yellow	18389	Au	IP2	7.673	7.673	7.673	7.673	7.673	7.673	8.02
5	Blue	18336	Au	IP2	14.27	14.27	14.27	14.27	14.27	191.43	14.27
6	Blue	18336	Au	IP2	9.715	9.715	12.612	12.612	9.715	146.96	12.612
7	Blue	18286	Au	IP2	230.11	259.39	230.11	256.79	230.15	230.18	230.11
8	Blue	18286	Au	IP2	187.92	187.92	187.92	187.92	251.96	188.35	187.92
9	Yellow	18286	Au	IP2	155.36	144.99	144.99	155.36	145.03	155.42	144.99
10	Yellow	18286	Au	IP2	33.277	36.879	36.879	36.879	48.153	33.278	36.879

Experimental Results

- Check repeat-ability of this technique using Cases 1-3 and Cases 5-6.
 - Some of these parameters are repeatable, but others are not.
- Caveats:
 - Case 1, the Booster was firing interfering with the tune-meter measurements
 - Case 3, the solution with the smallest χ^2 was not chosen, because b was negative.
 - This opens the question: How do we choose the solution?

	Case 1		Case 2		Case 3		Case 5		Case 6	
	Value	$\pm\sigma$	Value	$\pm\sigma$	Value	$\pm\sigma$	Value	$\pm\sigma$	Value	$\pm\sigma$
α_1	0.146	0.020	0.147	0.023	0.119	0.038	0.0797	0.021	0.0923	0.015
β_1	8.176	0.086	8.438	0.110	8.419	0.170	9.455	0.120	9.479	0.091
α_2	-0.029	0.015	-0.0079	0.017	-0.0109	0.031	0.0199	0.048	-0.0401	0.016
β_2	8.929	0.094	9.163	0.122	8.914	0.187	9.946	0.339	9.616	0.109
a	0.100	0.0081	0.102	0.0079	0.0963	0.0117	-0.0081	0.016	-0.0189	0.0077
b	0.705	0.117	0.972	0.115	1.085	0.160	1.268	0.098	1.348	0.080
c	0.0091	0.0019	-0.0074	0.0015	-0.0088	0.0022	-0.0129	0.0015	-0.0165	0.0014
d	0.0311	0.010	0.0236	0.0088	0.0135	0.0109	-0.0552	0.0082	-0.0612	0.0066
χ^2	3.511		8.624		16.478		14.270		9.715	

Conclusion

- Presented a method for measuring β^* in a coupled machine
- Tested automation, many bug fixes and improvements during APEX
 - Works in simulation
 - Need to improve repeat-ability
 - Need to test when RHIC is coupled
- Questions
 - Model accuracy required?
 - How good are the tunes and ΔQ_{min} measurements?
 - Are the optimizers good enough?

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